GAS-DYNAMIC CALCULATION OF PULSATING FLOW IN PIPELINES

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A numerical solution is examined for a system of equations of one-dimensional isothermal flow of a perfect gas in a horizontal pipe with a periodically varying function of the flow rate at the boundary. The numerical solution is compared with the solution of the linearized problem. The results can be used to calculate the pulsating motion of gas in the pipeline systems of piston compressors [1].

The one-dimensional isothermal motion of a perfect gas in a horizontal cylindrical pipe is described by a system of quasilinear hyperbolic equations [2]:

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho W)=0 \tag{1}$$

$$\frac{\partial}{\partial t}(\rho W)+\frac{\partial}{\partial x}(P+\rho W^2)+\frac{\lambda}{2D}\rho W|W|=0 \tag{2}$$

$$P=\rho C^2$$

$$0<x<L,\ t>0 \tag{3}$$

Here x is the coordinate along the axis of the pipe, t is the time, P, W, and ρ are the pressure, velocity, and density of the gas averaged over the cross section, D is the diameter of the pipe, λ is the coefficient of friction, and C is the velocity of sound.

A stationary distribution of gas pressure and velocity along the length of the pipe is set as the initial conditions:

$$P(x,0)=P_0=\text{const},\ W(x,0)=W_0=\text{const},\ 0\leqslant x\leqslant L \tag{4}$$

The neglect of pressure losses due to friction in the initial conditions for the concrete example considered below proves to have no significant effect on the characteristics of flow for times long enough after the start and possessing practical interest. The boundary conditions have the form

$$W(0,t)=W_0+W_*\sin\omega t,\ P(L,t)=P_0 \tag{5}$$

The solution of system (1)-(3) for the conditions (4) and (5) was determined by the numerical method of finite differences (method of grids). A characteristic form of writing (1)-(3) was used for the numerical calculation:

$$\frac{\partial P}{\partial t}+(W\pm C)\frac{\partial P}{\partial x}\pm\rho C\left[\frac{\partial W}{\partial t}+(W\pm C)\frac{\partial W}{\partial x}\right]=\pm C\Phi \tag{6}$$

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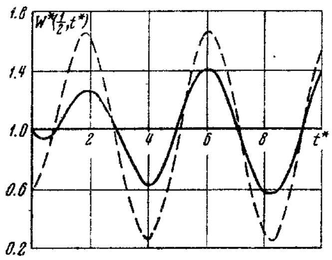

Fig. 1

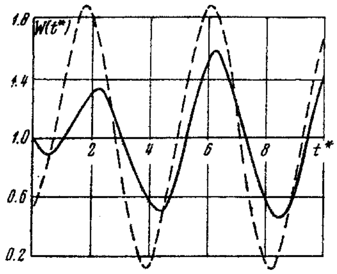

Fig. 2

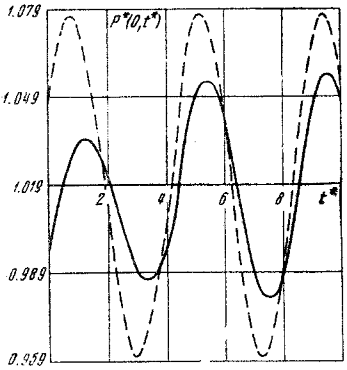

Fig. 3

$$\Phi = -\rho \frac{\lambda}{2D} W|W|$$

The system of difference equations for the internal points of a section i = 1, 2, ..., N−1 had the form

$$\begin{gathered}(P_{i,k+1} - P_{i,k})/\tau + (W_{i,k} \pm C)(P_{i+1,k+1} - P_{i-1,k+1})/2h \pm \\ \pm (\rho C)_{i,k}[(W_{i,k+1} - W_{i,k})/\tau + \\ + (W_{i,k} \pm C)(W_{i+1,k+1} - W_{i-1,k+1})/2h] = \pm C\Phi_{i,k}\end{gathered} \tag{7}$$

where h and τ are the steps of an orthogonal grid with respect to the x coordinate and the time t, respectively. Additional finite-difference functions, approximating the corresponding equations of system (6) in the vicinity of the boundary, were used in determining the solution at the boundary points. The boundary difference equation for the point i = 0 has the form

$$\begin{gathered}(P_{0,k+1} - P_{0,k})/\tau + (W_{0,k} - C)(P_{1,k+1} - P_{0,k+1})/h - \\ - (\rho C)_{0,k}[(W_{0,k+1} - W_{0,k})/\tau + (W_{0,k} - C)(W_{1,k+1} - W_{0,k+1})/h] = -C\Phi_{0,k}\end{gathered} \tag{8}$$

Correspondingly for the point i = N

$$\begin{gathered}(P_{N,k+1} - P_{N,k})/\tau + (W_{N,k} + C)(P_{N,k+1} - P_{N-1,k+1})/h + \\ + (\rho C)_{N,k}[(W_{N,k+1} - W_{N,k})/\tau + \\ + (W_{N,k} + C)(W_{N,k+1} - W_{N-1,k+1})/h] = C\Phi_{N,k}\end{gathered} \tag{9}$$

An analogous implicit difference system for solving a quasilinear system of hyperbolic equations has been examined by a number of authors [3-6].

The system of difference equations obtained together with the boundary conditions (5) form a closed system of linear algebraic equations, for whose solution a matrix method was used. The preliminary introduction of the variables

$$x^* = \frac{x}{L}, \quad t^* = \frac{Ct}{L}, \quad P^* = \frac{P}{P_0}, \quad W^* = \frac{W}{W_0} \tag{10}$$

$$V^* = \frac{W_*}{W_0}, \quad M = \frac{W_0}{C}, \quad H = \frac{\omega L}{C}, \quad R = \frac{\lambda}{2}\frac{L}{D}$$

of the original system led to a dimensionless form.

The following numerical values of the parameters were chosen for the example under consideration: L = 20 m, C = 315 m/sec, W_0 = 20 m/sec, $P_0 = 5 \cdot 10^4$ kg/m^2, W_* = 5 m/sec, ω = 23 sec^{-1}, D = 0.05 m, λ = 0.02, and the steps of the grid were h^* = 0.05 and τ^* = 0.15, respectively.

The dependence on the time t^* of the gas velocity at the middle and end points and of the pressure at the start and middle of the pipe are shown by solid lines in Figs. 1-4. The graphs are constructed on the basis of calculations carried out using a BÉSM-6 electronic computer. The convergence of the numerical solution was studied practically by comparison with solutions obtained for variations in the steps of the

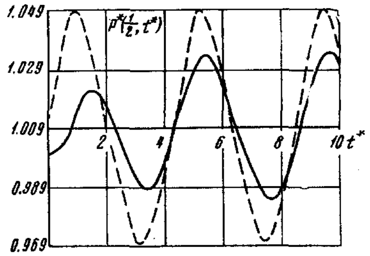

Fig. 4

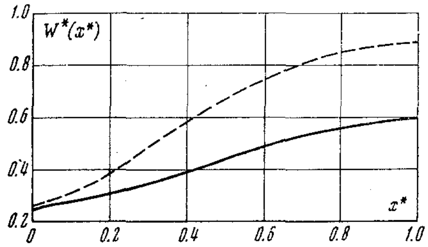

Fig. 5

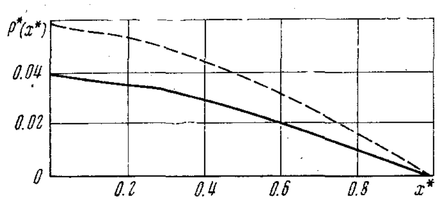

Fig. 6

orthogonal grid. In this case a twofold decrease in the grid parameters compared with those chosen leads to a practically insignificant deviation in the pressure and velocity values. An analysis of the data obtained showed that on any section $0 \le x^* \le 1$ for $t^* > 10$ the dependence of P^* and W^* on time becomes periodic, with the oscillation frequency practically coinciding with the driving frequency.

In connection with the fact that at present approximative methods [1, 2, 7–9] based on linearization of the original system (1)–(3) are used to solve problems of the analysis of pulsating movement of gas in pipelines, the problem of estimating their errors is of interest. According to the results obtained in [7, 8], using linearization by means of a representation of the desired flow parameters in the form of a sum of stationary and small nonstationary components, for a stationary oscillatory process with the boundary conditions (5), the solution of the linearized system (1)–(3) can be presented in the form

$$W^*(x^*, t^*) = 1 + \frac{V^*}{\operatorname{ch} 2MR + \cos 2H}\{\sin Ht^* [\cos Hx^* \operatorname{ch} MR(2 - x^*) + \\ + \cos H(2 - x^*) \operatorname{ch} MRx^*] - \cos Ht^* [\sin Hx^* \operatorname{sh} MR(2 - x^*) + \sin H(2 - x^*) \operatorname{sh} MRx^*]\} \tag{11}$$

$$P^*(x^*, t^*) = 1 + RM^2(1 - x^*) + \frac{V^*M}{\operatorname{ch} 2MR + \cos 2H}\{\sin Ht^* [\cos Hx^* \operatorname{sh} MR \times \\ \times (2 - x^*) - \cos H(2 - x^*) \operatorname{sh} MRx^*] + \cos Ht^* [\sin H(2 - x^*) \operatorname{ch} MRx^* - \sin Hx^* \operatorname{ch} MR(2 - x^*)]\} \tag{12}$$

In Figs. 1–4 the dashed curves represent solutions of the linearized system (1)–(3) at the corresponding points obtained with the use of these relations. The comparision shows that after completion of the transient processes ($t^* > 10$) the solutions of the nonlinear and linearized systems become very close in frequency and phase of the oscillations but can differ considerably in amplitude.

The lengthwise distributions of oscillation amplitudes of velocity and pressure for the nonlinear and linearized systems are represented by solid and dashed curves, respectively, in Figs. 5 and 6. Since the divergence of the amplitudes reaches values on the order of 50%, linearization can substantially show up the precision in determining the characteristics of the oscillatory process.

Additional studies are necessary to establish the exact limits of the ratio of W_* to W_0 within which it is possible to linearize the system (1)–(3). However, because this ratio is always greater than 0.25 in real pipeline systems of piston compressors, one can be sure that with the use of a linearized system for analyzing gas-dynamic processes in pipelines the results obtained are very rough.

The problem of calculating the pulsating movement of gas in a concrete pipeline system of a piston compressor differs from that examined in that the solutions of system (1)–(3) for several sections must be pieced together using the boundary conditions. The numerical method described above with the corresponding variables can also be used for this case.

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